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A DIRECT TEST OF PERTURBATIVE QCD AT SMALL x

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Abstract

We show that recent data from HERA on the proton structure function F_2 at small x and large Q^2 provide a direct confirmation of the double asymptotic scaling prediction of perturbative QCD. A linear rise of $\ln F_2$ with the scaling variable σ is observed throughout the kinematic region probed at HERA, and the measured slope is in excellent agreement with the QCD prediction. This provides a direct determination of the leading coefficient of the beta function. At large values of the scaling variable ρ the data display a small but statistically significant scaling violation.

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Perturbative QCD predicts that at sufficiently large $t \equiv \ln Q^2/\Lambda^2$ and small x the nucleon structure function F_2 should exhibit double scaling in the two variables

$$\sigma \equiv \sqrt{\ln \tfrac{x_0}{x} \ln \tfrac{t}{t_0}}, \qquad \rho \equiv \sqrt{\ln \tfrac{x_0}{x} / \ln \tfrac{t}{t_0}}, \tag{1}$$

provided only that the nonperturbative input to the perturbative evolution is sufficiently soft. We have shown [1] that this prediction is indeed confirmed by the first measurements of F_2^p performed at HERA[2,3]. In fact, it turns out that not only most of the HERA data, but even some of the older data from the NMC [4], lie well inside the asymptotic regime, suggesting that the starting scale $t_0 \equiv \ln Q_0^2/\Lambda^2$ for the perturbative evolution should be little more than $Q_0^2 \sim 1$ GeV^2. A significantly enlarged set of measurements of F_2^p has now become available [5,6], which makes it possible to test double scaling more quantitatively. Specifically, the slope of the linear rise of $\ln F_2$ in the scaling variable σ can be reliably measured, and turns out to be in excellent agreement with the QCD prediction, thus giving a direct empirical determination of the leading coefficient β_0 of the QCD beta–function. We also find that there is now evidence for scaling violation at large ρ.

Double asymptotic scaling follows from a computation [7] of the asymptotic form of the structure function $F_2^p(x;t)$ at small x based on the use of the operator product expansion and renormalization group at leading perturbative order. It thus relies only on the assumption that any increase in $F_2^p(x;t)$ at small x is generated by perturbative QCD evolution, rather than being due to some other (nonperturbative) mechanism manifested by an increase in the starting distribution $F_2^p(x;t_0)$. The resulting asymptotic behaviour takes the form

$$F_2^p(\sigma,\rho) \sim N f(\tfrac{\gamma}{\rho}) \tfrac{\gamma}{\rho} \tfrac{1}{\sqrt{\gamma\sigma}} \exp\left[2\gamma\sigma - \delta(\tfrac{\sigma}{\rho})\right] \left[1 + O(\tfrac{1}{\sigma})\right], \tag{2}$$

where $\gamma \equiv 2\sqrt{N_c/\beta_0}$, $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$, $\delta \equiv (1 + \frac{2n_f}{11N_c^3})/(1 - \frac{2n_f}{11N_c})$, and the unknown function f, which depends on the details of the starting distribution, tends to one for sufficiently small values of its argument. N is an a priori undetermined normalization factor.

In [1] we derived (2) by noting that at small-x the one loop QCD evolution equations reduce to wave equations, which propagate the parton distribution functions from their boundary values at $t = t_0$ and $x = x_0$ to larger values of t and smaller values of x. Since the propagation is unstable, away from the boundaries an exponential increase with σ of the form (2) inevitably arises, provided only that the small-x behaviour of the starting

distributions at t_0 is sufficiently soft (which in practice means that if $f_s(x;t)$ is a singlet parton distribution function, $x^{1+\lambda} f_s(x;t_0) \to 0$ as $x \to 0$ for any $\lambda \lesssim 0.2$). The behaviour (2) is thus a rather clean prediction of perturbative QCD, in so far as it is independent of the details of the (soft) nonperturbative parton distributions which are input at t_0, provided that at small x these conform to expectations based on Regge theory. The asymptotic behaviour can be shown [1] to set in rather rapidly as σ increases in a region not too close to the boundaries, i.e. when ρ is neither too large nor too small.

In order to compare the data for F_2^p with the prediction (2) we rescale the measured values of F_2 by a factor

$$R'_F(\sigma,\rho) = R \exp\left(\delta(\sigma/\rho) + \tfrac{1}{2}\ln\sigma + \ln(\rho/\gamma)\right), \tag{3}$$

to remove the part of the leading subasymptotic behaviour which can be calculated in a model independent way.[1] Then $\ln[R'_F F_2]$ is predicted to rise linearly with σ, independently of ρ (when ρ is large), with slope

$$2\gamma = 12/\sqrt{33 - 6n_f/N_c} = 2.4 \tag{4}$$

if $n_f = 4$ as in the HERA kinematic range. The model-dependent subasymptotic behaviour due to the function f can be eliminated by cutting all points with subasymptotically small ρ; the scaling analysis of Ref.[1] (see fig. 2 below) suggests that we place the cut at $\rho^2 = 2$.

All the available experimental data[4,5,6] for F_2^p which pass this cut are plotted in fig. 1. The predicted linear rise in σ is spectacularly confirmed, providing clear evidence that in the region $\sigma^2 > 1$, $\rho^2 > 2$ the asymptotic behaviour (2) has set in. Indeed, the scaling actually sets in rather precociously: even the NMC data down to $\sigma \sim 0.7$ seem to be rising linearly, with possibly an indication of a systematic normalization mismatch of around 10% between the NMC and the HERA determinations of F_2.

Fitting a straight line to all 80 HERA points in the plot yields a χ^2 of 66, and a gradient $2\gamma_{exp} = 2.37 \pm 0.16$, in perfect agreement with the QCD prediction eq.(4). Turning this into a measurement of the leading coefficient of the beta–function gives (with $N_c = 3$) $\beta_0 = 8.6 \pm 1.1$ (to be compared with 25/3 for $n_f = 4$). This is a direct, model independent, and highly nontrivial test of the perturbative dynamics of asymptotically free nonabelian gauge theory.

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[1] The constant rescaling factor R may of course be chosen arbitrarily; here we choose $R = 8.1$, so that the normalization of the figures is the same as in [1].

	N_s	N_h	χ^2
a)	0.341 ± 0.005	0	96
b)	0	0.156 ± 0.002	878
a)+b)	0.319 ± 0.012	0.012 ± 0.002	91

Table: The fitted normalizations N_s and N_h and the associated χ^2s (103 data points). The different cases considered are a) soft pomeron b) hard pomeron, and the linear combination a) + b).

We next consider scaling violations, both in the subasymptotic region of small σ and small ρ, and in the post-asymptotic region of large ρ. This is best done by rescaling F_2^p by a factor

$$R_F(\sigma,\rho) = R \exp\big(-2\gamma\sigma + \delta(\sigma/\rho) + \tfrac{1}{2}\ln\sigma + \ln(\rho/\gamma)\big) \tag{5}$$

to remove all the leading behaviour in (2). The rescaled structure function should thus scale in both σ and ρ when both are sufficiently large to lie in the asymptotic region: $R_F F_2^p = N + O(1/\sigma) + O(1/\rho)$. This double asymptotic scaling behaviour is tested in the two scaling plots fig. 2, where we also display the predictions obtained [1] by applying the leading small-x form of the evolution equations to a typical soft starting gluon distribution. Specifically, fig. 2a) shows that the scaling in σ sets in very rapidly, as all the points on the plot lie in the asymptotic regime; fig. 2b) shows that the scaling in ρ only sets in for $\rho^2 \gtrsim 2$. However even if ρ is as low as $\rho \sim \frac{1}{2}$ the subasymptotic corrections due to $f(\gamma/\rho)$ seem fairly well accounted for by the scaling violation displayed by the curves of fig. 2.

More interestingly, at large ρ there now appears to be a statistically significant rise above the scaling prediction. To test the significance of this rise, we fitted to the data a linear combination of the behaviour discussed above and displayed by the curves of fig. 2, and a "hard pomeron" behaviour, which violates scaling by rising with ρ (see ref.[1] for a more detailed discussion). Including in the fit the 103 HERA points with both σ^2 and ρ^2 greater than one half gives the results displayed in the table. The data seem to prefer a $4 \pm 1\%$ admixture of the hard pomeron solution. One should be very cautious about taking this as evidence for the hard pomeron per se, however, since higher loop corrections should give a similar rise[8]. It should be possible to settle this issue decisively when a more detailed set of data and more accurate theoretical calculations become available.

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Figure Captions

Fig. 1. Values of $R'_F F_2^p$ plotted against σ: diamonds are ZEUS data [5], squares H1 data [6], and crosses are NMC data. The best fit straight line is also shown.

Fig. 2. $R_F F_2^p$ plotted against a) σ and b) ρ. Included in the plots are all the HERA data with $\rho > 1.2$, $\sigma > 0.7$, respectively. The curves show the prediction obtained [1] evolving a typical soft starting gluon distribution: a) dot-dash curve, $\rho = 1.4$; solid curve, $\rho = 2.2$; dotted curve, $\rho = 3.2$. b) dot-dash curve, $\sigma = 1.1$; solid curve, $\sigma = 1.8$; dotted curve, $\sigma = 2.1$.

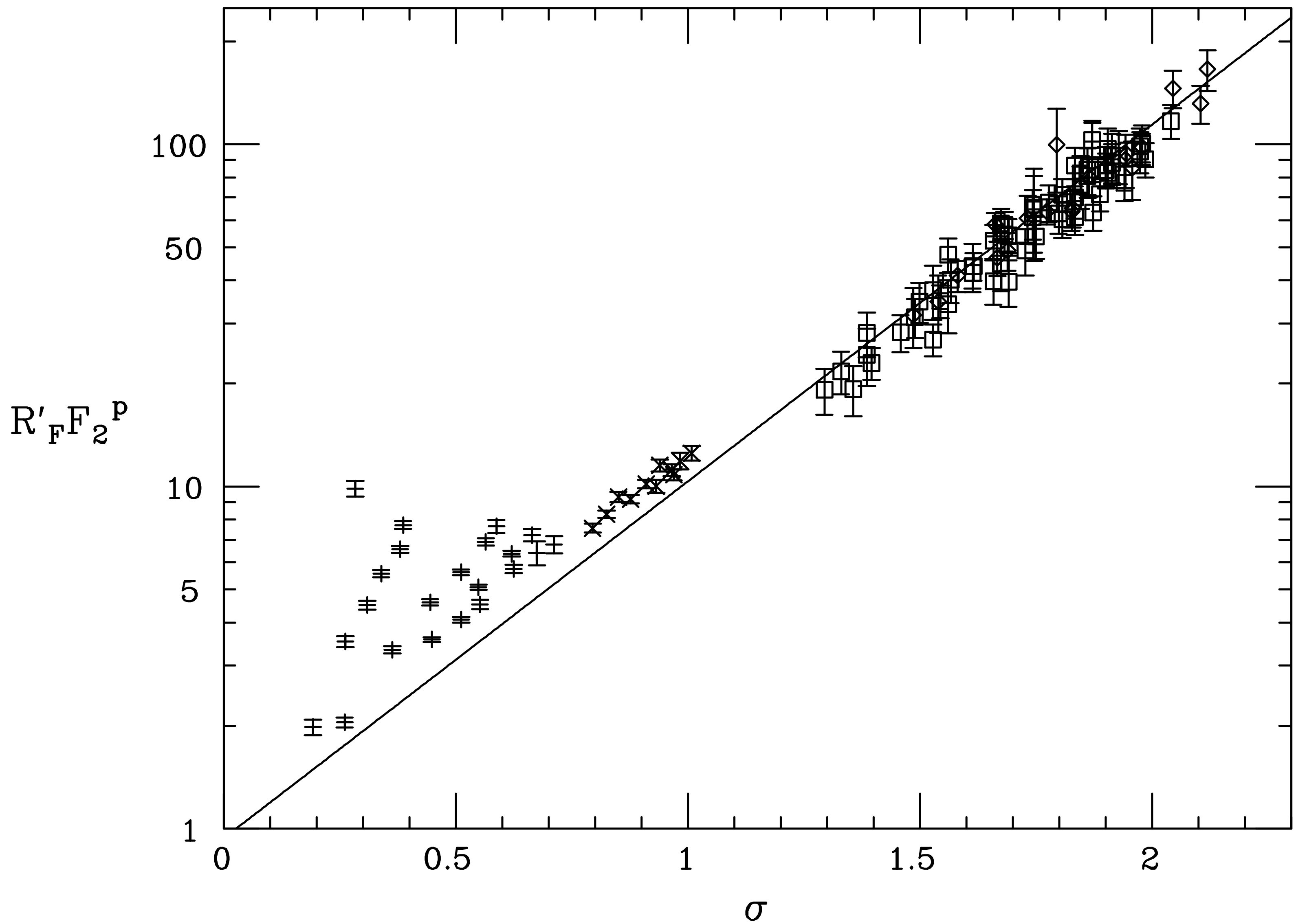

Fig. 1

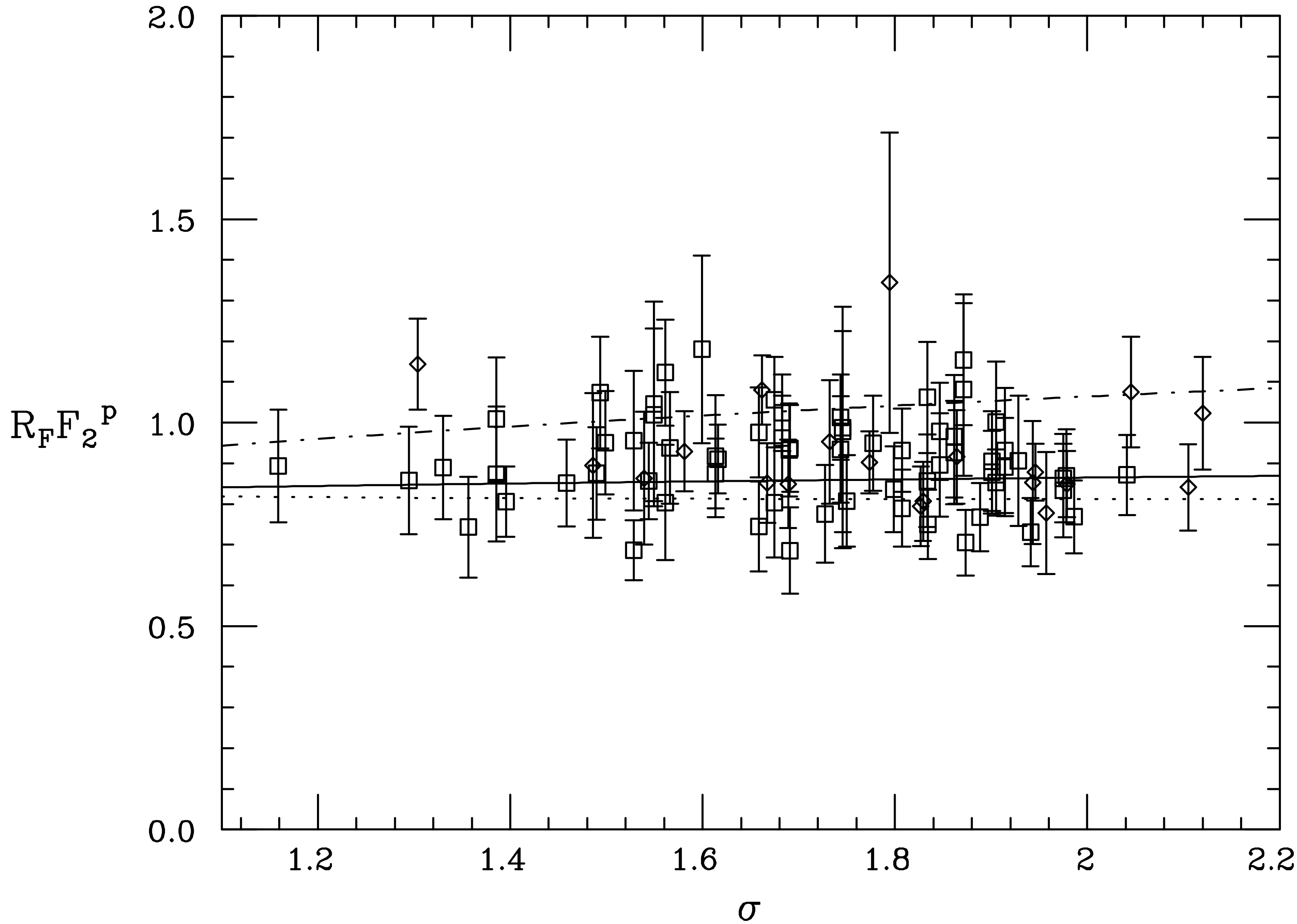

Fig. 2a

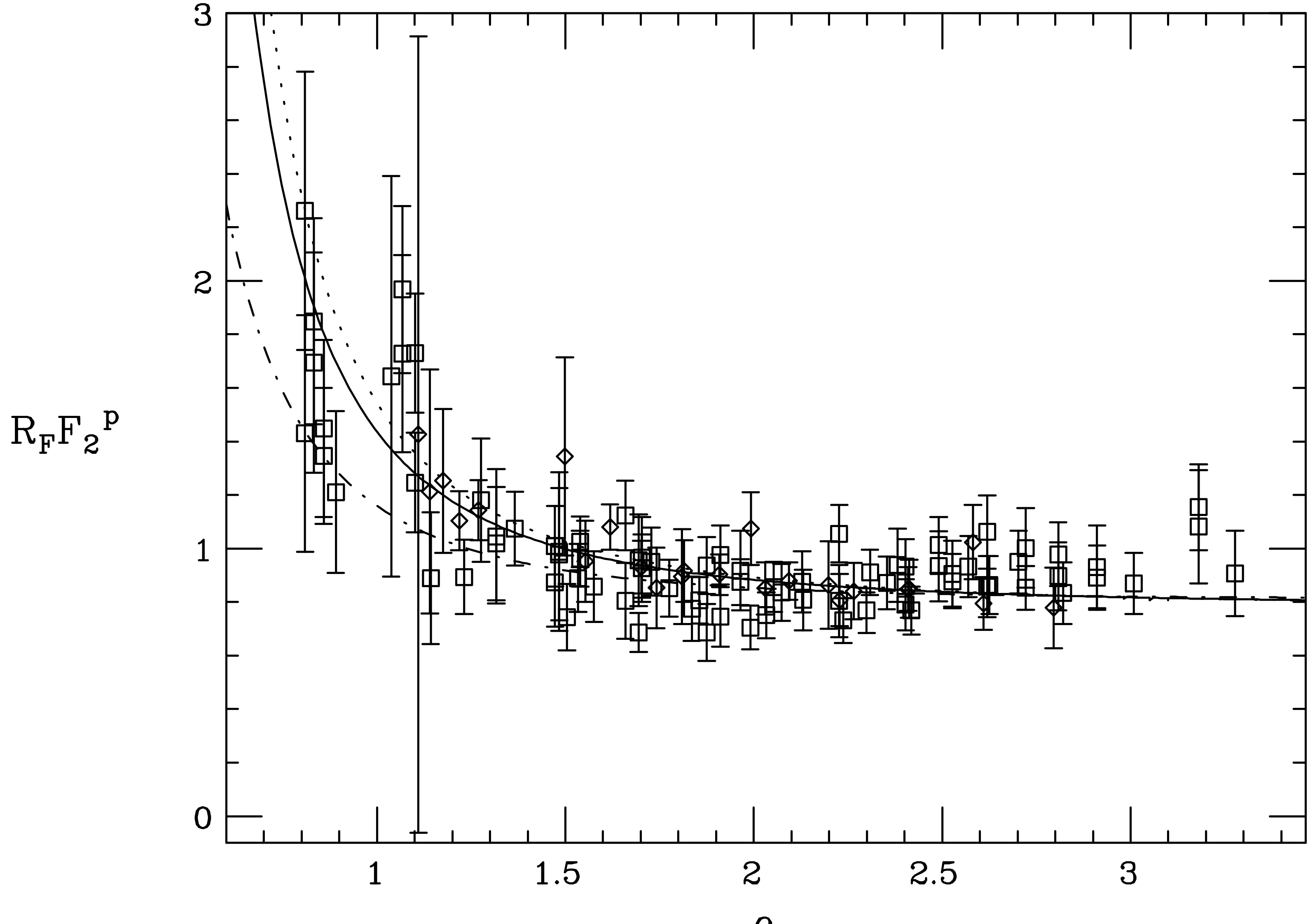

Fig. 2b